

Impact of Jets

1. The liquid comes out in the form of a jet from the outlet of a nozzle which is fitted to a pipe through which the liquid is flowing under pressure.

The following cases of the impact of jet, i.e. the force exerted by the jet on a plate will be considered:-

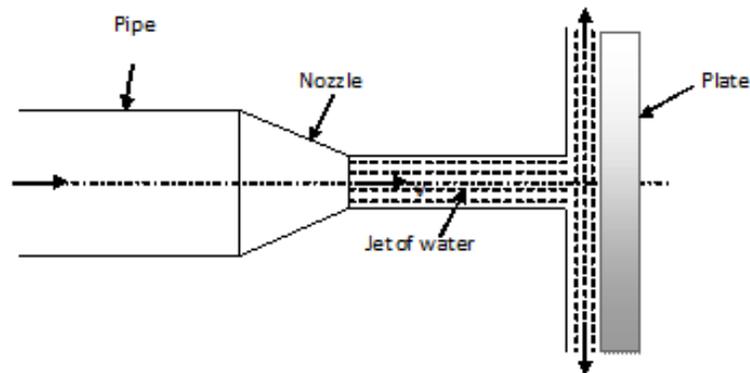
1. Force exerted by the jet on a stationary plate
 - a) Plate is vertical to the jet
 - b) Plate is inclined to the jet
 - c) Plate is curved

2. Force exerted by the jet on a moving plate

- a) Plate is vertical to the jet
- b) Plate is inclined to the jet
- c) Plate is curved

Force exerted by the jet on a stationary vertical plate

Consider a jet of water coming out from the nozzle strikes the vertical plate



V = velocity of jet, d = diameter of the jet, a = area of x – section of the jet

The force exerted by the jet on the plate in the direction of jet.

F_x = Rate of change of momentum in the direction of force

= initial momentum – final momentum / time

= mass x initial velocity – mass x final velocity / time

= mass/time (initial velocity – final velocity)

= mass/ sec x (velocity of jet before striking – final velocity of jet after striking)

= $\rho aV (V - 0)$

= ρaV^2

Force exerted by the jet on the moving plate

1st Case: Force on flat moving plate in the direction of jet

Consider, a jet of water strikes the flat moving plate moving with a uniform velocity away from the jet.

V = Velocity of jet

a = area of x-section of jet

U = velocity of flat plate

Relative velocity of jet w.r.t plate = $V - u$

Mass of water striking/ sec on the plate = $\rho a(V - u)$

Force exerted by jet on the moving plate in the direction of jet

$$F_x = \text{Mass of water striking/ sec} \times [\text{Initial velocity} - \text{Final velocity}]$$

$$= \rho a(V - u) [(V - u) - 0]$$

$$= F_x = \rho a(V - u)^2$$

In this case, work is done by the jet on the plate as the plate is moving,

for stationary plate the w.d is zero.

Work done by the jet on the flat moving plate

$$= \text{Force} \times \text{Distance in the direction of force} / \text{Time}$$

$$= W.d = \rho a(V - u)^2 \times u$$

2nd Case: Force on inclined plate moving in the direction of jet

Consider, a jet of water strikes the inclined plate moving in the direction of jet with a relative velocity.

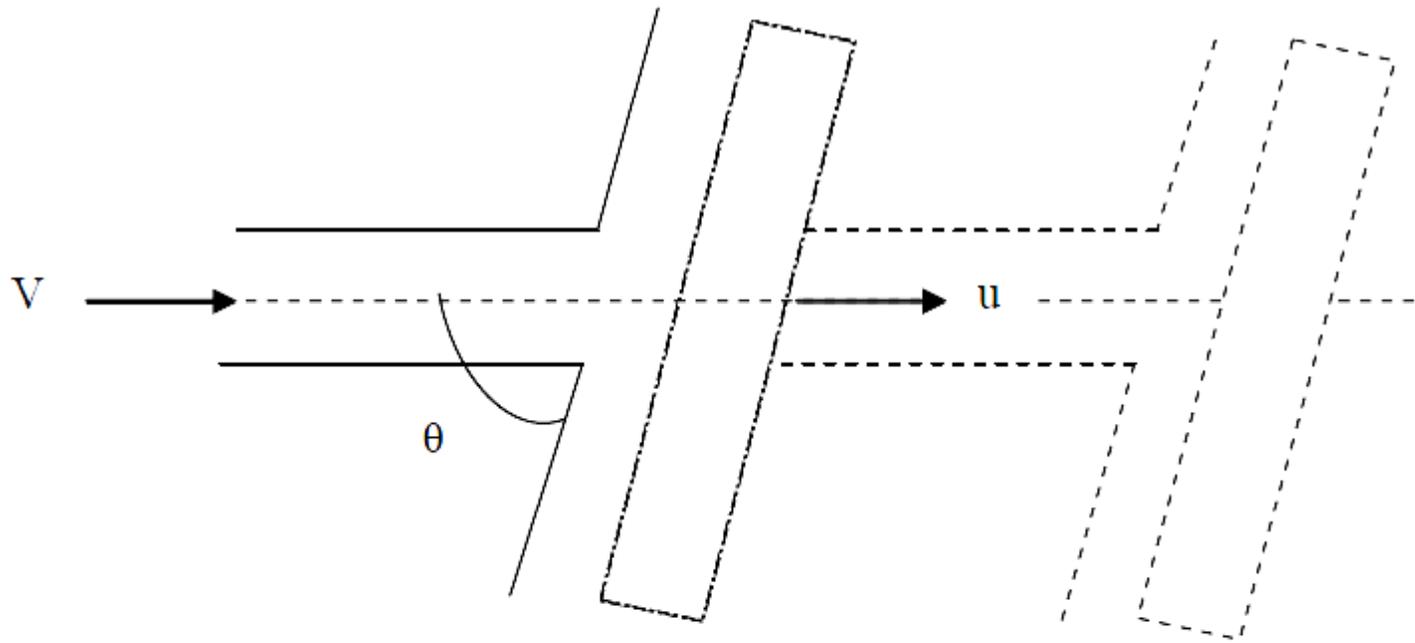
V = Velocity of jet

a = area of x-section of jet

U = velocity of flat plate

Relative velocity of jet w.r.t plate = $V - u$

If the plate is smooth, it is assumed that the loss of energy due to impact of jet is zero, then the jet of water leaves the inclined plate with a velocity $(V - u)$.



Force exerted by jet on the inclined plate in the direction normal to the jet

$$F_n = \text{Mass of water striking/ sec} \times [\text{Initial velocity} - \text{Final velocity}]$$

$$= \rho a(V - u) [(V - u) \sin\theta - 0]$$

$$F_n = \rho a(V - u)^2 \sin\theta$$

This normal force can be resolved into two components one in the direction of jet and other perpendicular to the direction of jet.

Component of F_n in the direction of jet.

$$F_x = \rho a(V - u)^2 \sin^2\theta$$

Component of F_n in the direction perpendicular to the direction of jet

$$F_y = \rho a(V - u)^2 \sin\theta \cos\theta$$

Work done by the jet on the flat moving plate

$$= \text{Force} \times \text{Distance in the direction of force} / \text{Time}$$

$$W.d = \rho a(V - u)^2 \sin^2\theta \times u$$

Force exerted by the jet of water on series of vanes

Force exerted by jet of water on single moving plate (Flat or curved) **is not feasible one, it is only theoretical one.**

Let,

V = Velocity of jet

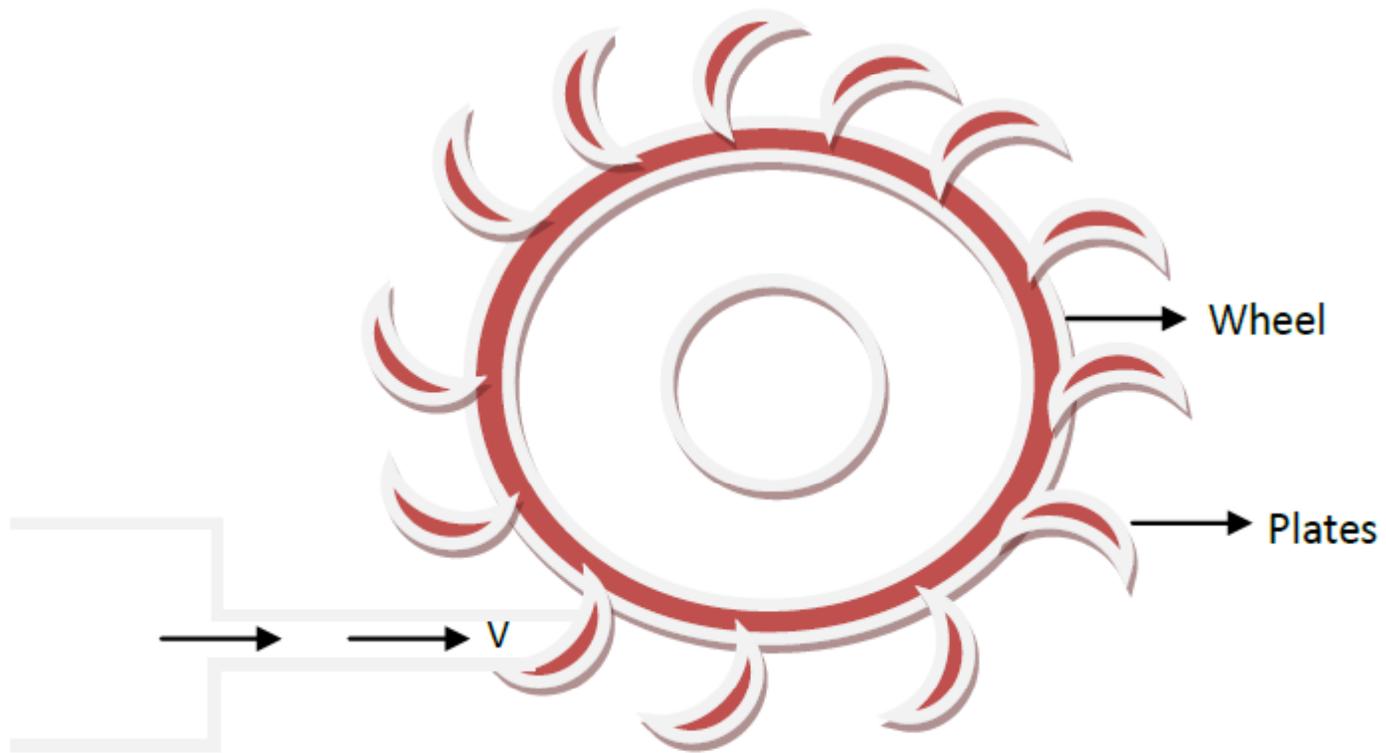
a = area of x-section of jet.

u = velocity of vane

In this, mass of water coming out from the nozzle/s is always in constant with plate. When all plates are considered.

Mass of water striking/s w.r.t plate = ρaV

Jet strikes the plate with a velocity = $V - u$



Force exerted by the jet on the plate in the direction of motion of plate
= Mass/sec x (Initial velocity – Final velocity)

$$F_x = \rho aV (V - u)$$

Work done by the jet on the series of plate/sec, Work Done = $F_x \times u$

$$\text{Work Done} = \rho aV (V - u) \times u$$

$$\begin{aligned} K.E \text{ of jet/sec} &= \frac{1}{2} mV^2 \\ &= \frac{1}{2} (\rho aV)V^2 \end{aligned}$$

$$K.E \text{ of jet/sec} = \frac{1}{2} \rho aV^3$$

$$\text{Efficiency, } \eta = \frac{\text{Work done by jet/s}}{K.E \text{ by jet/s}}$$

$$= \frac{\rho a V (V - u) \times u}{\frac{1}{2} (\rho a V) V^2}$$

$$\eta = \frac{2 u (V - u)}{V^2}$$

Condition for Max Efficiency,

Efficiency is maximum when, $\frac{d\eta}{du} = 0$

$$\frac{d}{du} \left[\frac{2 u (V - u)}{V^2} \right] = 0$$

$$\frac{d}{du} \left[\frac{2 u V - 2 u^2}{V^2} \right] = 0$$

$$\left[\frac{2 V - 4u}{V^2} \right] = 0$$

$$u = \frac{V}{2}$$

Put the values of u in $\eta = \frac{2 u (V - u)}{V^2}$

$$\eta_{max} = \frac{2 \frac{V}{2} (V - \frac{V}{2})}{V^2}$$

$$\eta_{max} = \frac{1}{2} = 50\%$$

Jet Propulsion

1. Jet propulsion means propulsion or movement of the bodies such as ships, rockets, aircrafts etc with the help of jet.
2. The **reaction of the jet** coming out from the nozzle attached to the bodies is used to move the bodies
3. A/c **Newton's third law of motion**, every action there is an equal and opposite reaction.
4. The fluid coming out from the nozzle exerts a force on the nozzle in the opposite direction.

Jet Propulsion of Ships

1. By the application of jet propulsion,
2. the ship is driven through a water.
3. jet of water is discharged at the back of ships, exerts a propulsive force on the ship.
4. The ship carries a centrifugal pump which draws a water from surrounding sea.
5. The water is discharged through the orifice which is fitted at the back of the ship in the form of jet.

Case-1 Jet propulsion of the ship, when inlet orifice is at right angle to the direction of motion of ship

V = velocity of jet coming out at the back of ship.

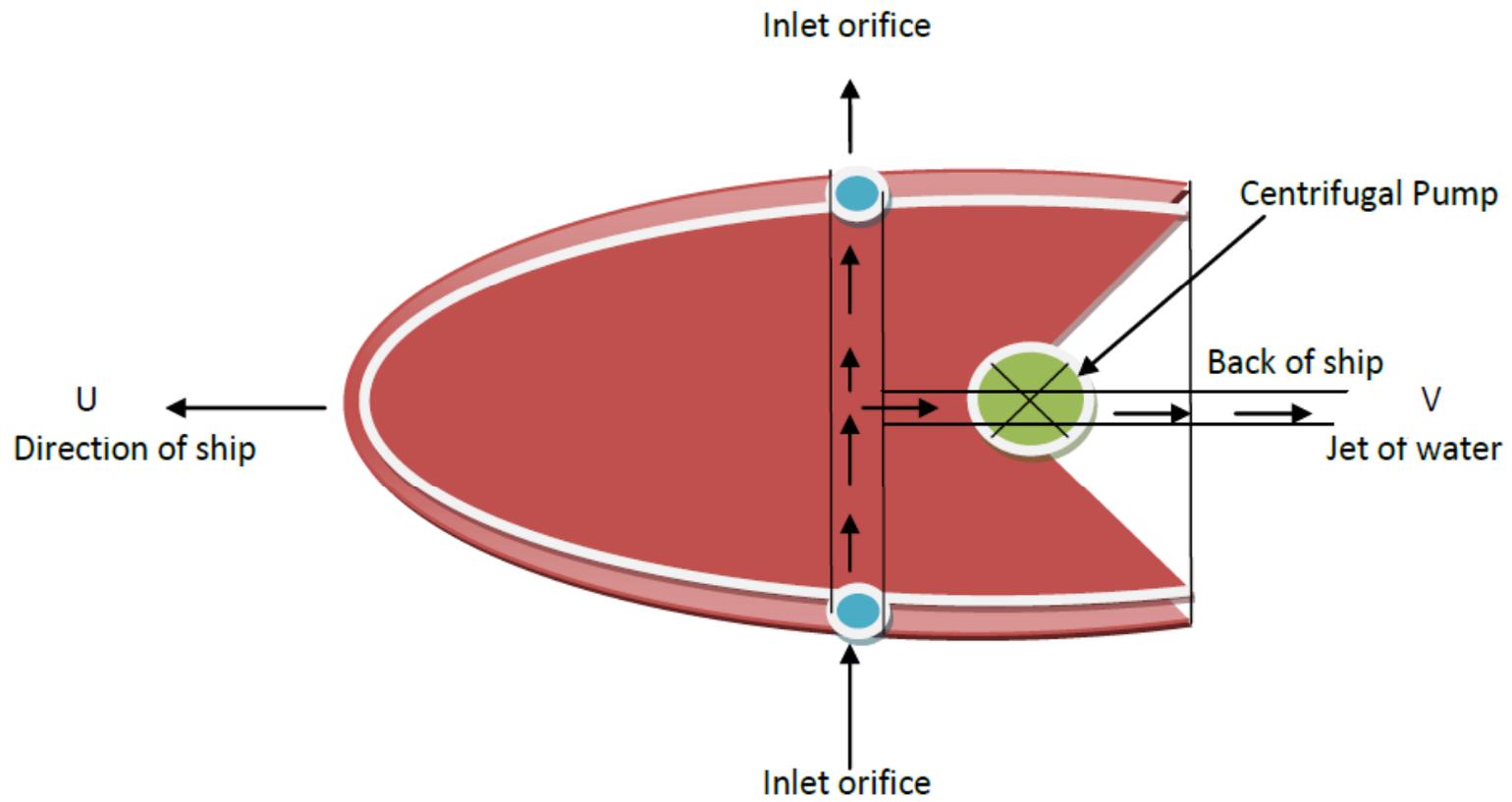
U = velocity of ship

= relative velocity of jet w.r.t ship

V and U are in opposite direction .

Relative velocity of jet w.r.t ship = $V + U$

Mass of jet issuing from nozzle at the back of ship = $\rho a (V + U)$



Force exerted by the jet on the ship = mass of water issuing/s x change in velocity

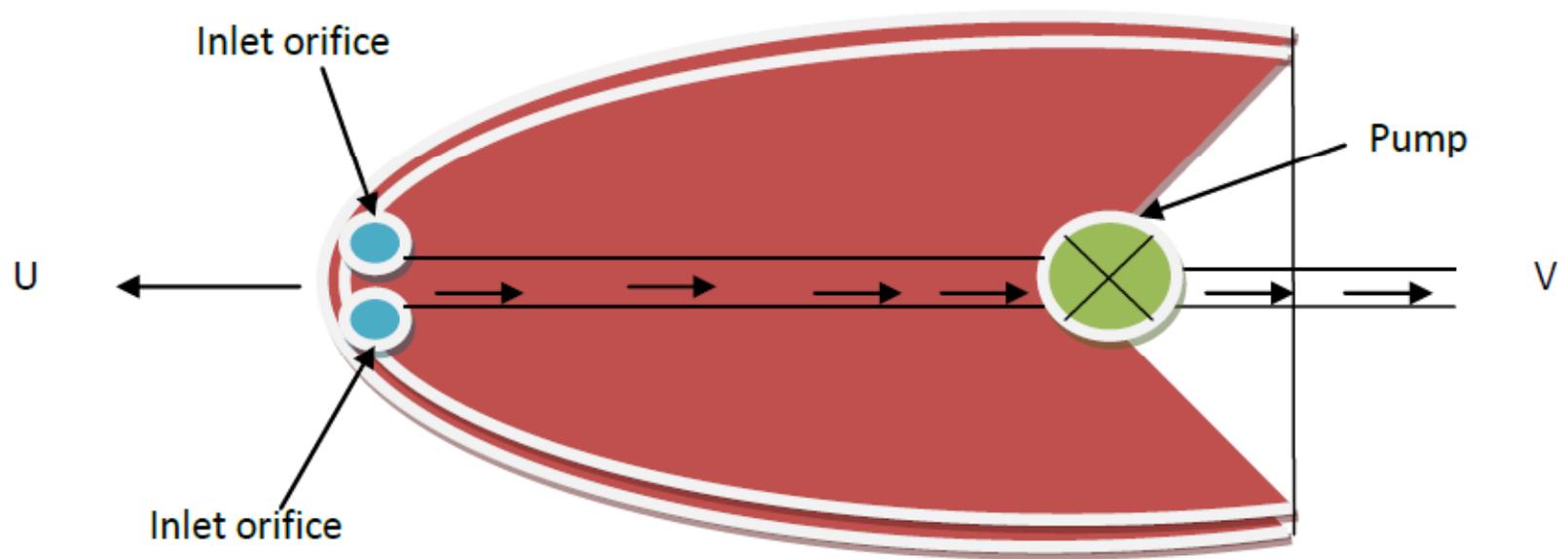
$$\begin{aligned} F &= \rho a (V + U) \times (V_r - u) \\ &= \rho a (V + U) \times ((V + U) - u) \\ F &= \rho a V (V + U) \end{aligned}$$

$$\begin{aligned} \text{Work done} &= F \times u \\ &= \rho a V (V + U) \times u \end{aligned}$$

Case-2 Jet propulsion of the ship, when inlet orifice facing the direction of motion of ship

Water enters the orifice with the same velocity as velocity of ship.

$$\begin{aligned} \text{Kinetic energy supplied by jet} &= \frac{1}{2} \times \text{mass of water} \times \text{change in velocity} \\ &= \frac{1}{2} \rho a V_r (V_r^2 - u^2) \end{aligned}$$



$$= \frac{1}{2} \rho a (V + u) ((V + u)^2 - u^2)$$

$$= \frac{1}{2} \rho a (V + u) (V^2 + 2uV)$$

$$\text{Efficiency of propulsion} = \frac{\text{work done by jet}}{\text{kinetic energy by jet}}$$

$$\text{Efficiency of propulsion} = \frac{\rho a (V + u) \times V \times u}{\frac{1}{2} \rho a (V + u) (V^2 + 2uV)}$$

$$\text{Efficiency of propulsion, } \eta = \frac{2u}{(V + 2u)}$$

Hydraulic Machines

1. Hydraulic machines are those machines which can convert either hydraulic energy into mechanical energy or from mechanical energy to hydraulic energy.
2. Hydraulic machines which converts hydraulic energy into mechanical energy are called turbines.

Classification of Hydraulic turbines

1. A/c to the type of energy available at the inlet
 - a) Impulse turbine
 - b) Reaction turbine
2. A/c to the direction of flow of runner
 - a) Tangential flow turbine
 - b) Axial flow turbine
 - c) Radial flow turbine
 - d) Mixed flow turbine

Pelton turbine

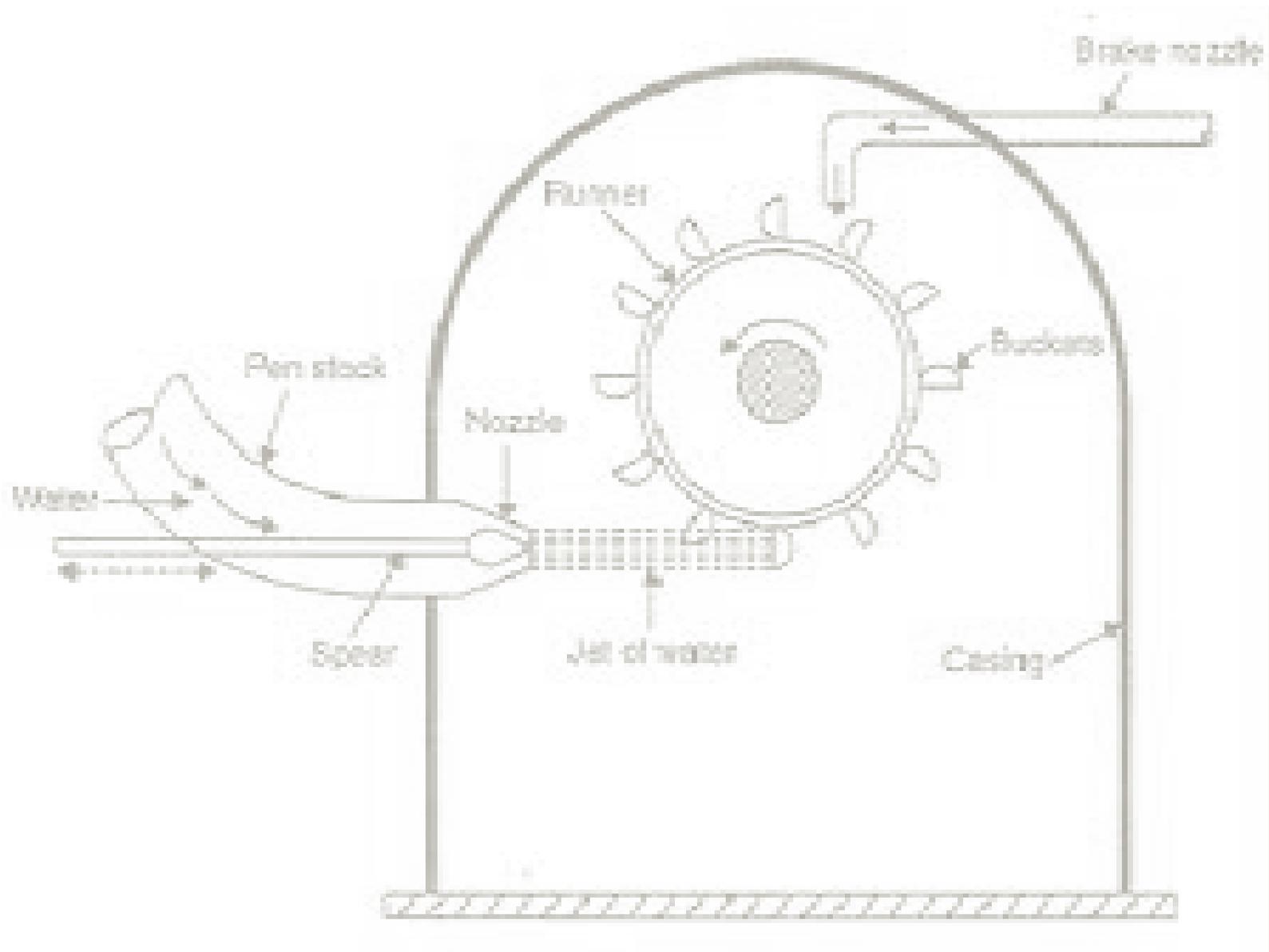
1. Pelton turbine is a **tangential flow impulse turbine**. In this the amount of water striking the bucket along the tangent of the runner.
2. The amount of **energy available at inlet** of turbine is **K.E.**
3. **Pressure at the inlet and outlet** of the turbine is **atmospheric**.

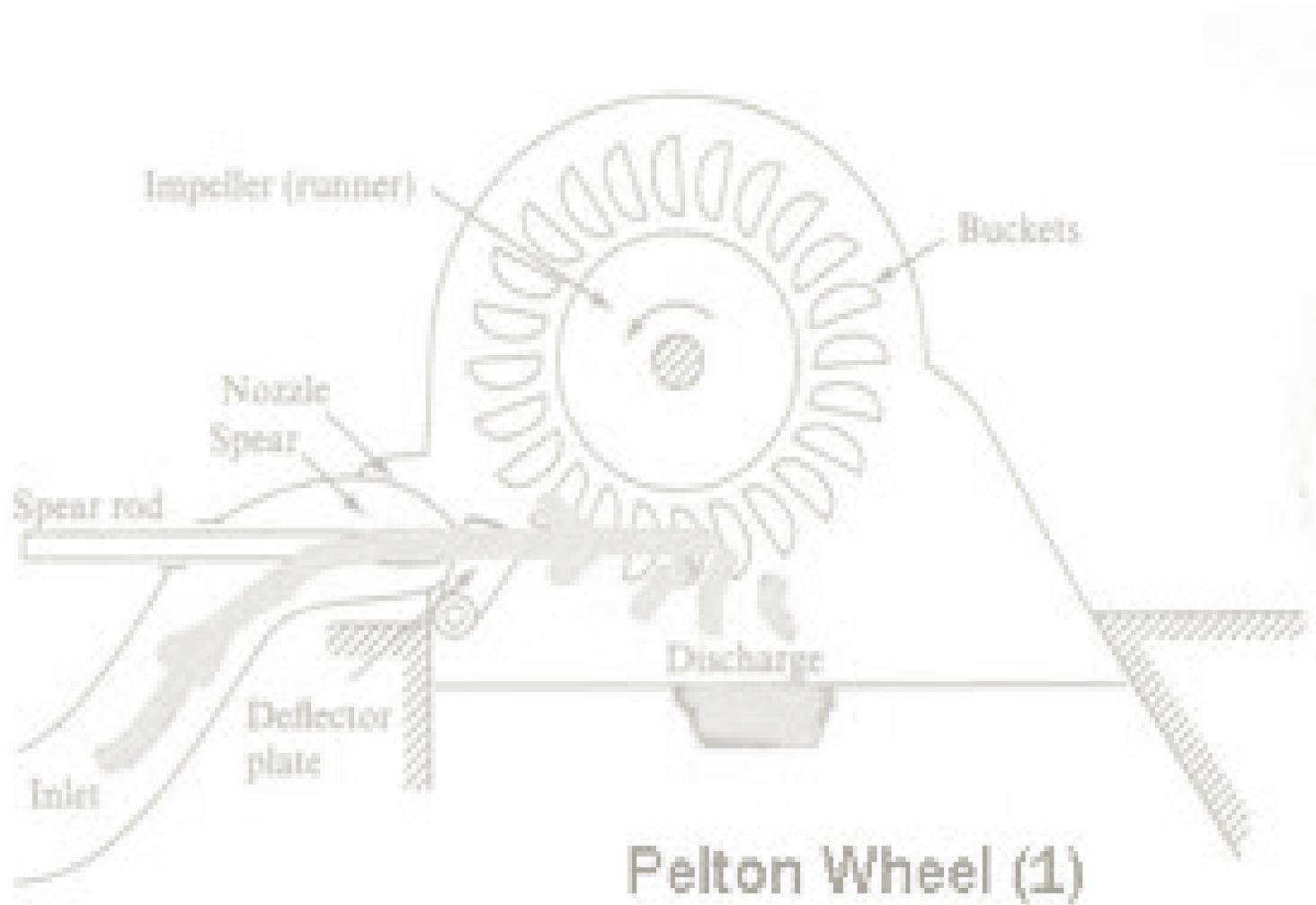
Main parts of Pelton turbine

1. Nozzle and flow regulating arrangements
2. Runner with buckets
3. Casing
4. Breaking jet

1. Nozzle and flow regulating arrangements

In this, amount of water striking the bucket of runner is controlled by providing a spear in the nozzle.





1. **Spear** is a conical needle which is operated either by **hand wheel** or **automatically** in the axial direction of flow depending upon the **size of the unit**.
1. When the **spear is move back**, the amount of water striking the runner is **increased**.

2. Runner with buckets

1. It consists of a **conical disk** on the periphery of which a **number of buckets** are fixed at some distance apart.
1. The **shape of the buckets** of the runner is **double-hemi spherical cup or bowl**.
2. The buckets are designed in such a way that the jet get deflected through **160° or 170°**.

3. Casing

1. The main function of casing is to **prevent the splashing of water and discharge of water to the tail race.**
2. It also provides **protection against accidents.** The casing of pelton wheel does not perform any hydraulic function.

4. Breaking jet

1. When the nozzle is completely closed by moving the spear in the forward direction, the amount of water striking the runner is reduced.
2. The runner because of **inertia** goes on revolving for the long time.

Velocity Triangle and Work done for Pelton wheel

The jet of water from the nozzle strikes the bucket at the splitter, which split the jet into two equal parts. These parts of jets glide over the inlet surface and comes out in the outer edge.

H = Net head acting on the Pelton wheel.

$$= H_g - h_f$$

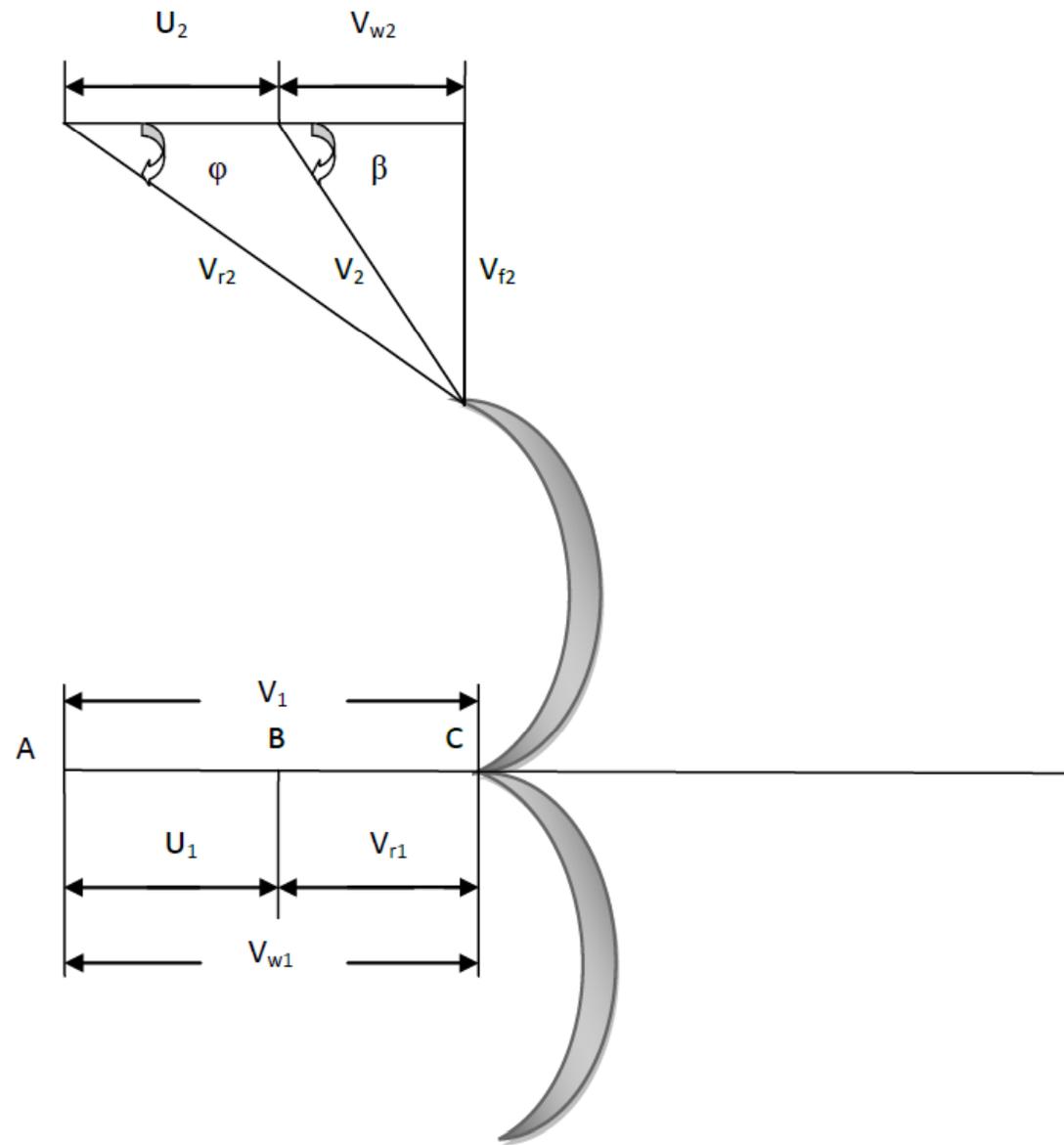
H_g = Gross head and h_f = head loss due to friction

$$h_f = \frac{4fLV^2}{D^* \times 2g}, \quad \text{where } D^* = \text{diameter of penstoke}$$

D = diameter of wheel, N = speed of wheel in rpm. d = diameter of jet,

$$V_1 = \text{Velocity of jet at inlet} = \sqrt{2gH}$$

$$u = u_1 = u_2 = \frac{\pi DN}{60}$$



The velocity Δ at inlet will be a straight line,

$$V_{w1} = V_1$$

$$V_{r1} = V_1 - u_1 = V_1 - u$$

The velocity Δ at outlet,

$$V_{r1} = V_{r2}, \quad V_{w2} = V_{r2} \cos \phi - u_2$$

The force exerted by jet of water in the direction of motion,

$$F_x = \rho a V_1 [V_{w1} + V_{w2}]$$

$$\text{Mass of water striking} = \rho a V_1$$

$$\text{Workdone by the jet on the runner / sec} = F_x \times u = \rho a V_1 [V_{w1} + V_{w2}] \times u, \quad Nm/s$$

$$\text{Power given to the runner by jet} = \frac{\rho a V_1 [V_{w1} + V_{w2}] \times u}{1000}, \quad KW$$

$$\begin{aligned}
 \text{Workdone/s per unit weight of water} &= \frac{\rho a V_1 [V_{w1} + V_{w2}] x u}{\text{Weight of water striking/s}} \\
 &= \frac{\rho a V_1 [V_{w1} + V_{w2}] x u}{\rho a V_1 x g} \\
 &= \frac{[V_{w1} + V_{w2}] x u}{g}
 \end{aligned}$$

$$\text{Hydraulic efficiency, } \eta_h = \frac{\text{Workdone/s}}{\text{K.E of jet/s}}$$

$$\text{K.E of jet/s} = \frac{1}{2} m V^2$$

$$= \frac{1}{2} \rho a V_1 (V_1^2)$$

$$= \frac{1}{2} \rho a V_1^3$$

$$\text{Hydraulic efficiency, } \eta_h = \frac{\rho a V_1 [V_{w1} + V_{w2}] x u}{\frac{1}{2} \rho a V_1^3}$$

$$\eta_h = \frac{2[V_{w1} + V_{w2}] x u}{V_1^2}$$

$$\eta_h = \frac{[V_1 + (V_1 - u) \cos \phi - u] x u}{V_1^2}$$

$$\eta_h = \frac{2(V_1 - u) (1 + \cos \phi) x u}{V_1^2}$$

The efficiency is maximum when $\frac{d(\eta_h)}{du} = 0$

$$\frac{d}{du} \left[\frac{2(V_1 - u) (1 + \cos \phi) x u}{V_1^2} \right] = 0$$

$$\frac{(1 + \cos \phi)}{V_1^2} \frac{d}{du} [2u(V_1 - u)] = 0$$

$$\frac{d}{du} [2uV_1 - 2u^2] = 0$$

$$u = \frac{V_1}{2}$$

Expression for maximum efficiency of pelton wheel

$$\eta_{h_{\max}} = \frac{2(V_1 - \frac{V_1}{2}) (1 + \cos \phi) \times \frac{V_1}{2}}{V_1^2}$$

$$\eta_{h_{\max}} = \frac{(1 + \cos \phi)}{2}$$

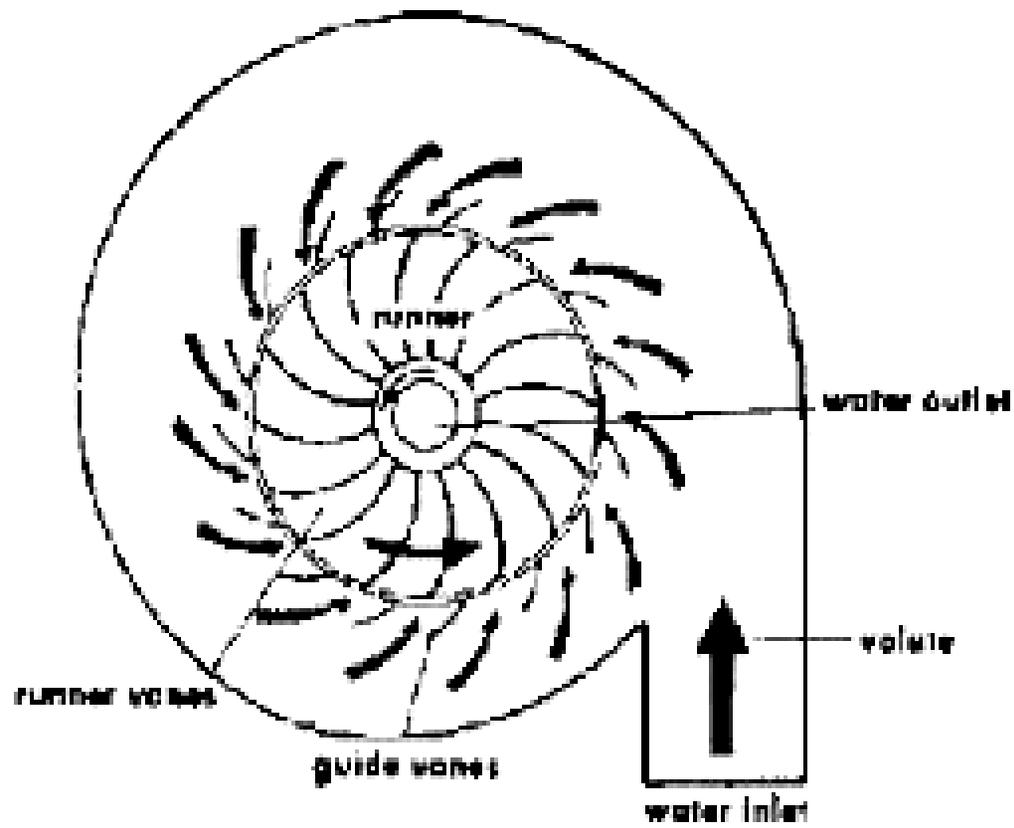
Design of Pelton wheel

Design of the Pelton wheel means following data is to be determine:-

- a) Diameter of the jet (d)
- b) Diameter of wheel (D) using $\frac{\pi DN}{60}$
- c) Width of bucket = 5 x d
- d) Depth of the bucket = 1.2 x d
- e) No of buckets on the wheel = **total rate of flow** through the turbine/ **rate of flow** of water through a single jet.

Radial flow reaction turbine

1. If the water is flow from outward to inward, then such turbine are called **inward flow radial turbine**.
2. If the water is flow from inward to outward then such turbine are called **outward flow radial turbine**.



Reaction turbine:-

1. These turbine are those in which the water at the inlet of the turbine possesses both **pressure energy** as well as **kinetic energy**.
1. The runner is completely enclosed in an air tight casing. The casing and the runner is always full of water.

Main parts of radial flow reaction turbine

- a) Casing
- b) Guide mechanism
- c) Runner
- d) Draft tube

Casing

1. In case of **reaction turbine**, casing and runner is always **filled with water**.
2. The casing may be of spiral shape because the water may enter into the runner at **constant velocity**.

Guide Mechanism

1. The **stationary guide vanes** are fixed on the guide mechanism .
2. The vanes can allow the water to strike on the vanes which are fixed on the runner.

Runner

1. It is a **circular wheel** on which a series of **guide vanes** are fixed. The **surface of the vanes are very smooth**,

.

Draft tube

1. The **pressure at the exit** of the turbine is **less than** atmospheric pressure.
2. The water is **not directly exit** and discharged through the runner.

Inward radial flow turbine

The water flows over the moving vanes in the **inward radial direction** and discharged the water at the **inner diameter of the runner**.

Work done /sec on the runner by the water given by

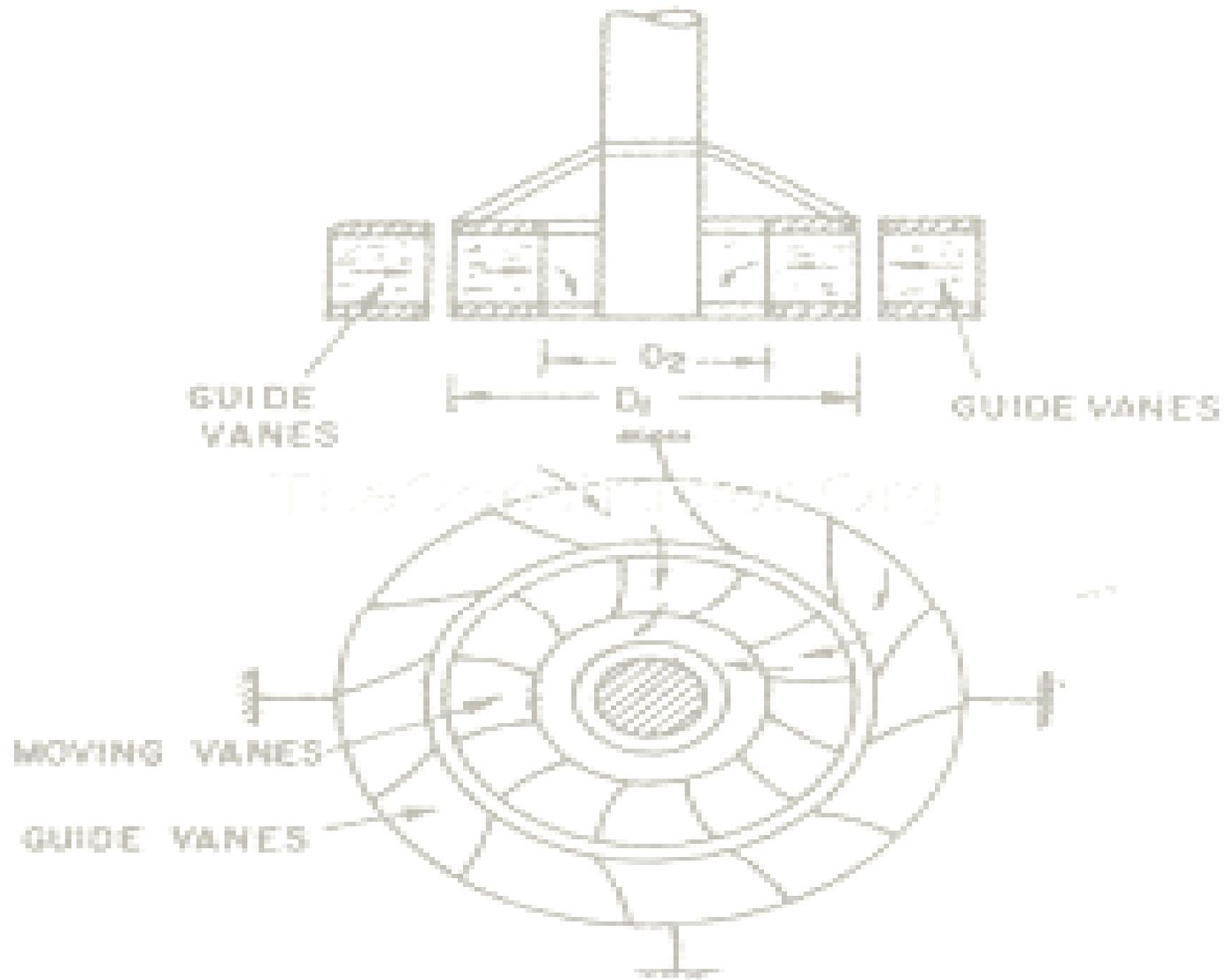
This equation represents the **energy transfer/s to the runner**

$$= \rho a V_1 [V_{w1} u_1 \pm V_{w2} u_2]$$

$$= \rho Q [V_{w1} u_1 \pm V_{w2} u_2]$$

$$u_1 = \text{tangential velocity of wheel at inlet} = \frac{\pi D_1 N}{60}$$

$$u_2 = \text{tangential velocity of wheel at outlet} = \frac{\pi D_2 N}{60}$$



$$\begin{aligned} \text{Workdone/s per unit weight of water} &= \frac{\text{workdone/s}}{\text{weight of water striking/s}} \\ &= \frac{\rho a V_1 [V_{w1} u_1 \pm V_{w2} u_2]}{\rho a V_1 x g} \\ &= \frac{[V_{w1} u_1 \pm V_{w2} u_2]}{g} \end{aligned}$$

Eulers equation of hydrodynamics machines

Note,

if β is an acute angle then +ve sign is used, when β is an obtuse angle then -ve sign is used.

If $\beta = 90^\circ$, $V_{w2} = 0$, then

$$\text{Workdone} = \frac{V_{w1} u_1}{g}$$

$$\text{Hydraulic efficiency} = \frac{\text{Runner power}}{\text{Water power}}$$

Runner power is the power developed/s by water to the runner.

$$\begin{aligned}\text{Hydraulic efficiency} &= \frac{\frac{W}{1000g} [V_{w1}u_1 \pm V_{w2}u_2]}{Wx H} \\ &= \frac{\frac{W}{1000g} [V_{w1}u_1 \pm V_{w2}u_2]}{Wx H} \\ &= \frac{[V_{w1}u_1 \pm V_{w2}u_2]}{Hg}\end{aligned}$$

If the discharge is radial at outlet, $V_{w2} = 0$,

$$\eta_H = \frac{V_{w1}u_1}{Hg}$$

Outward radial flow reaction turbine

1. In this case, water enters into the casing and enter into the stationary guide wheel.
2. The velocity triangle at inlet and outlet will be drawn by the same procedure.
3. But in this case, **inlet of the runner is at inner diameter of runner,**
4. **tangential velocity at inlet will be less than outlet of the turbine.**

$$U_1 < U_2 \text{ as } D_1 < D_2$$